Matrix product approximations to multipoint functions in two-dimensional conformal field theory

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Goal

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  - Tensor networks model the entanglement properties of many body systems and are successfully applied in condensed matter physics
  - What about quantum field theories?
- States of the quantum field theory and tensor network states live in different Hilbert spaces: how to measure closeness?

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- If tensor networks can approximately reproduce correlation functions of quantum field theories, then we can use them to understand the entanglement structure of quantum field theories.

- Start with simplest interesting class of quantum field theories: 1+1 dimensional unitary Conformal Field Theories (a quantum field theory defined on the circle with conformal symmetry)

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- Tensor network states for spin chains:

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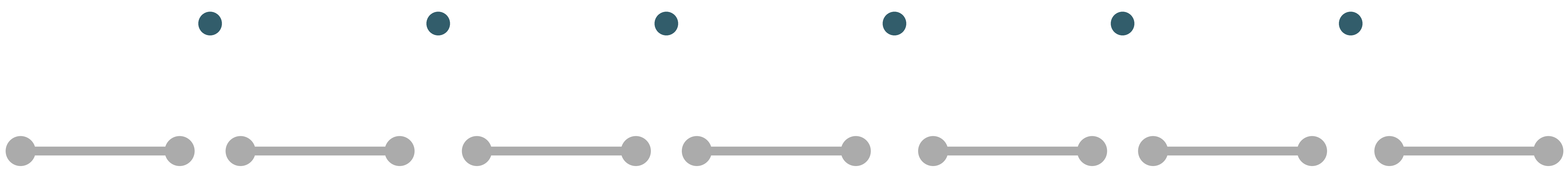

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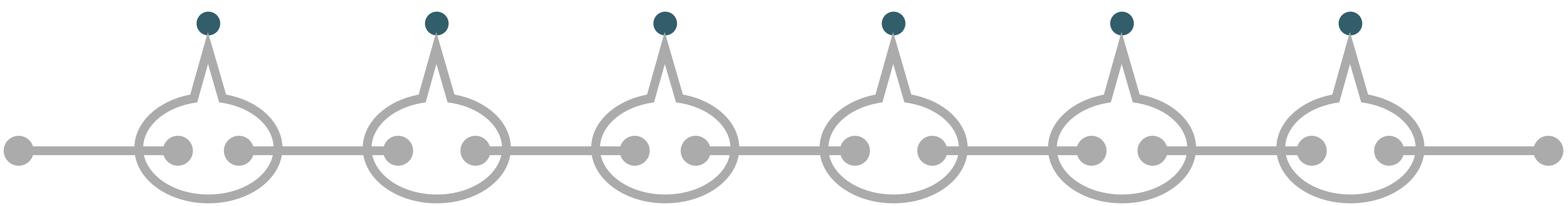
- Tensor network states for spin chains:

maximally entangled pairs of (bond) dimension D are placed between the physical particles (physical dimension d) and are contracted by a DxDxd dimensional tensor



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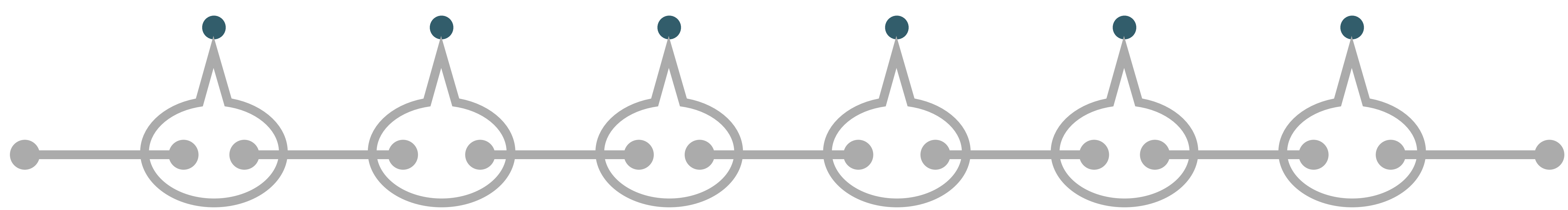

- Correlation functions can be computed efficiently (in D) and reduce to the computation of a sequence of completely positive maps on matrices of dimension D

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Scaling of Parameters:
number of fields n, UV cutoff d (measured in terms of energy), approximation error ε, C constant depending on CFT (not necessarily central charge)

scaling of bond dimension	fixed n, UV cutoff d	fixed ε
log(D)	$\sim \log(1/\varepsilon)\, C\, n/d$	$\sim \sqrt{C\, n}$

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- Proof is mathematically rigorous and constructive; holds for most unitary Conformal Field Theories
- However, the parameter scaling is worse than would could be expected from entropic arguments [Cardy&Calabrese, Holzhey et. al.,..]
- Uses the language of Vertex operator algebras: first introduced by Borcherds in his proof of the Moonshine conjecture

More Symmetries: Wess-Zumino-Witten models

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- These additional symmetries carry over to the MPS Tensors; leads to a group invariant MPS

- Moreover, the interactions (fusion rules) are completely described already in the lowest level; the higher order Tensors are only needed to model the conformal and affine symmetries

Proof sketch: regularization

- identify states with Hilbert-Schmidt operators on the chiral theory; field operators become linear maps: need to approximate by finite-dimensional ones

$$\langle \phi_1(x_1) \quad \phi_2(x_2) \quad \ldots \quad \phi_n(x_n) \rangle$$

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- techniques: use results of Wassermann for WZW models (explicit bounds), and the existence of genus-1 correlation functions for general CFTs [Zhu, Huang]

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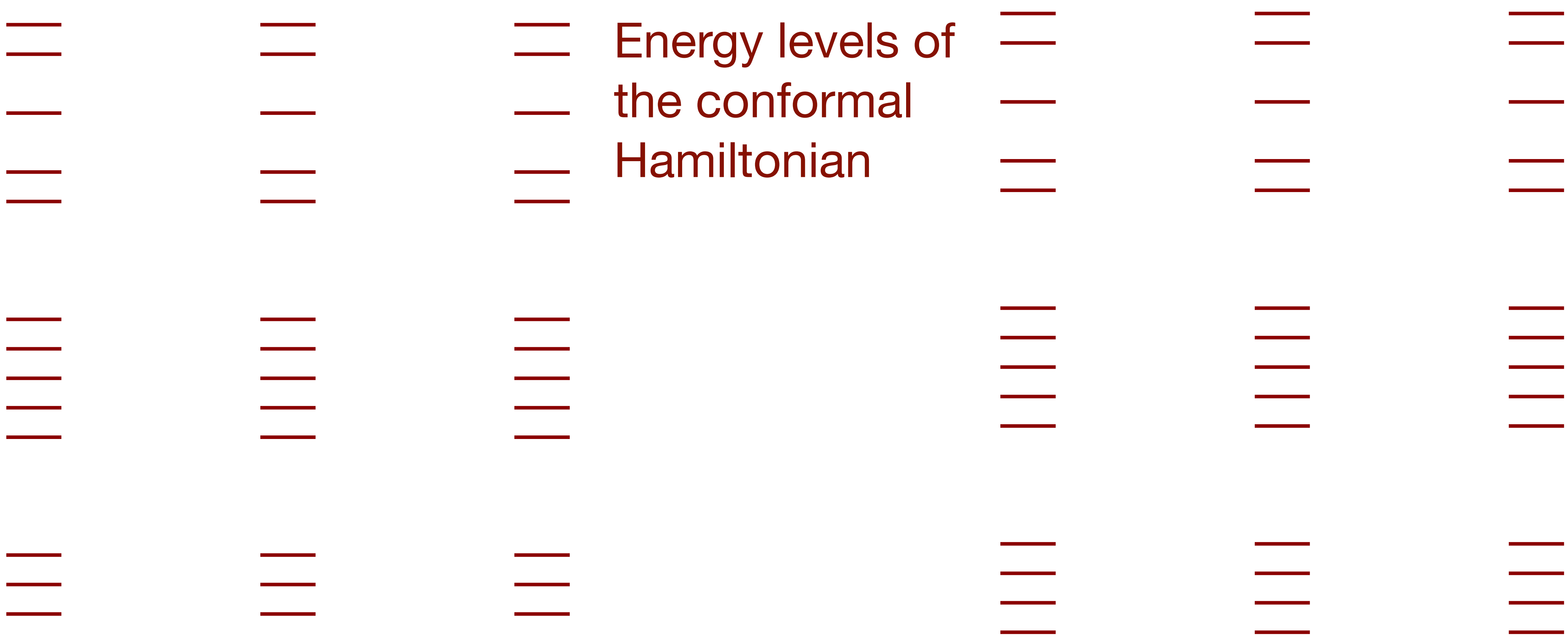
Bounded field operator φ(x): can change the energy by an arbitrary amount



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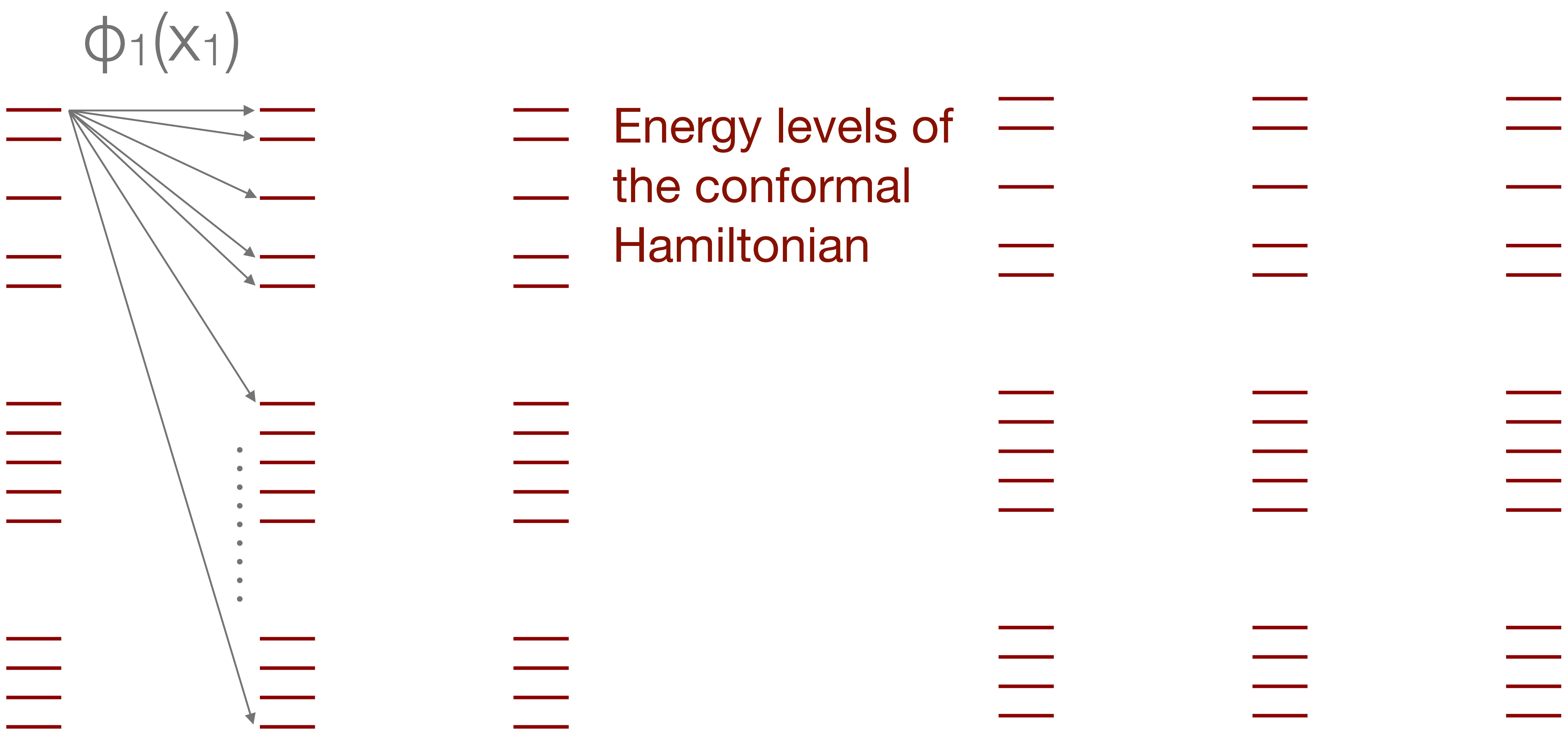

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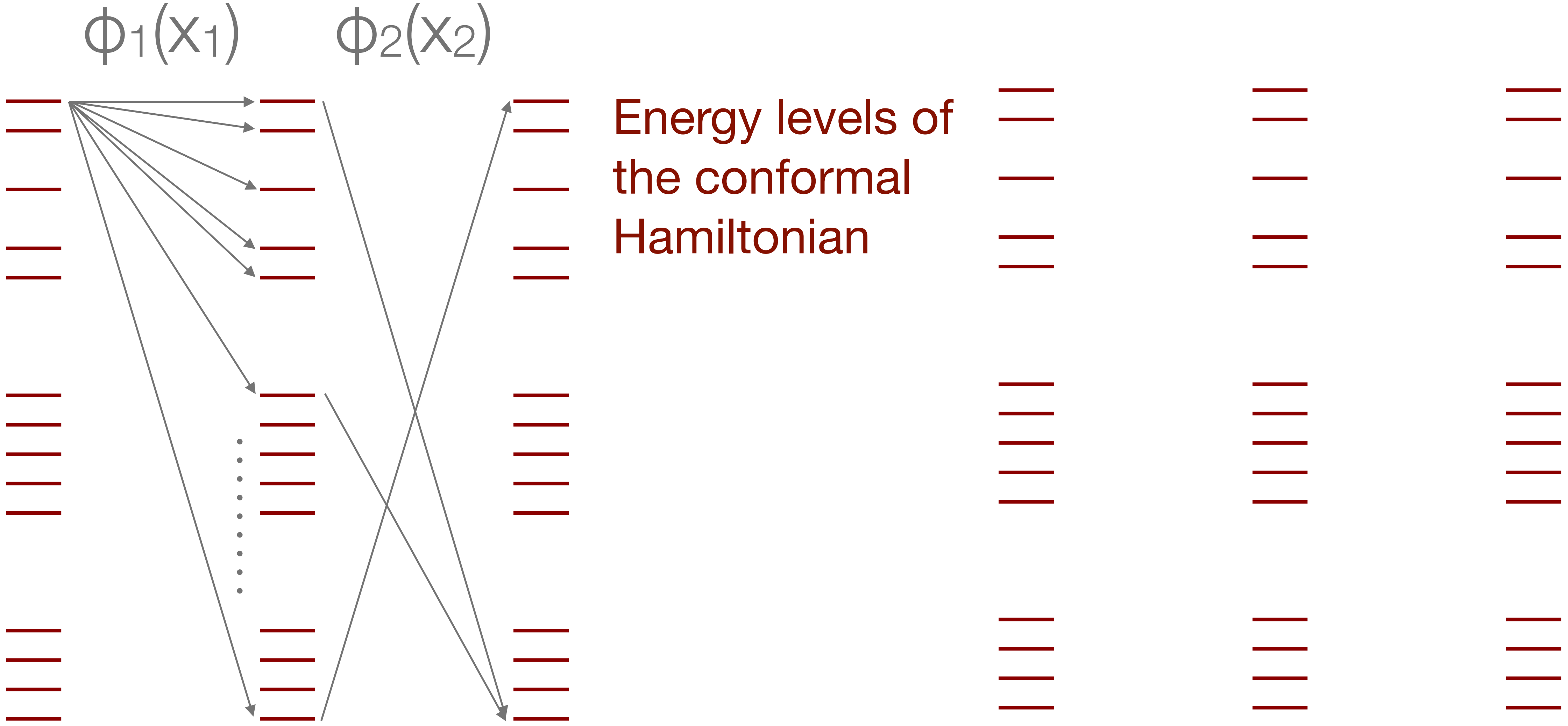

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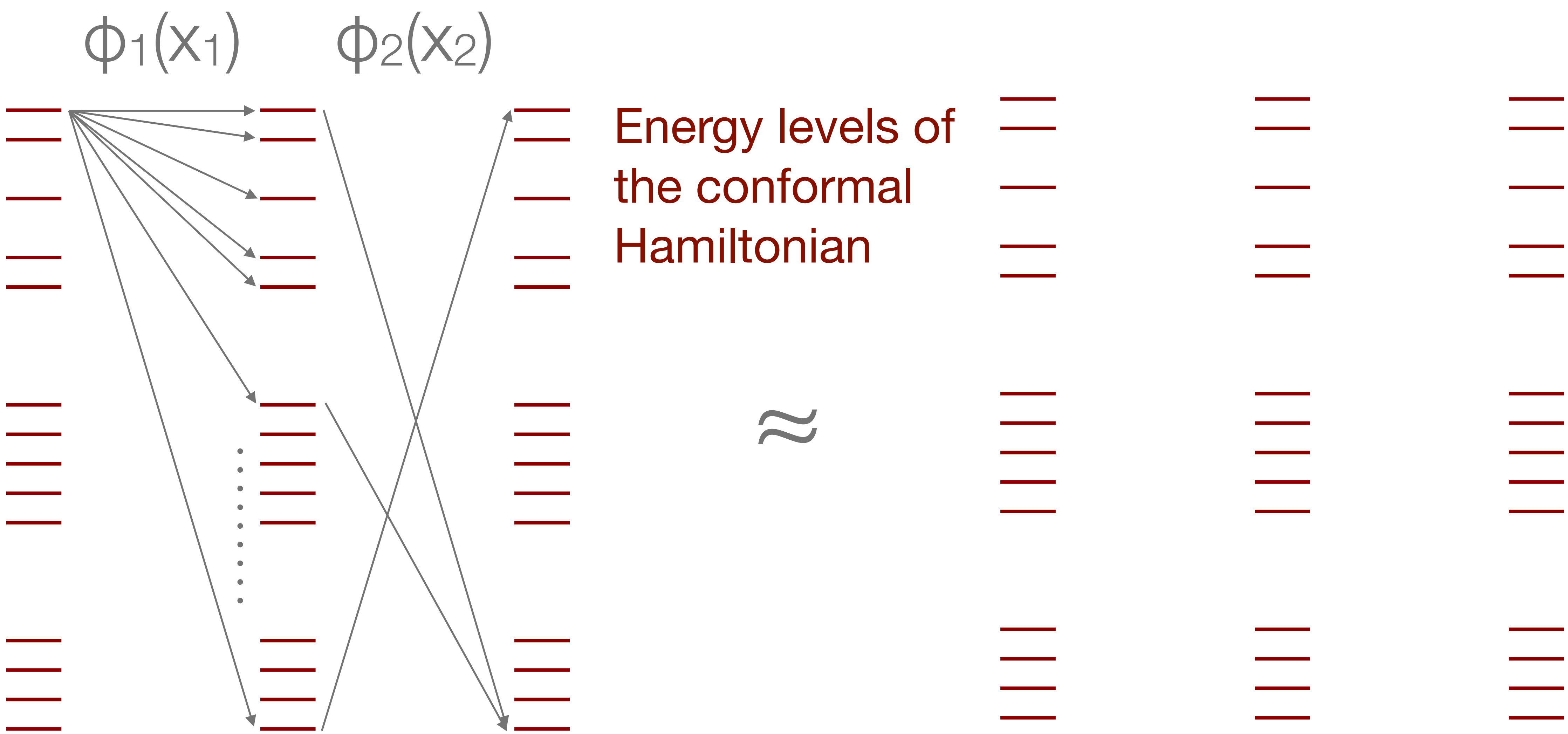

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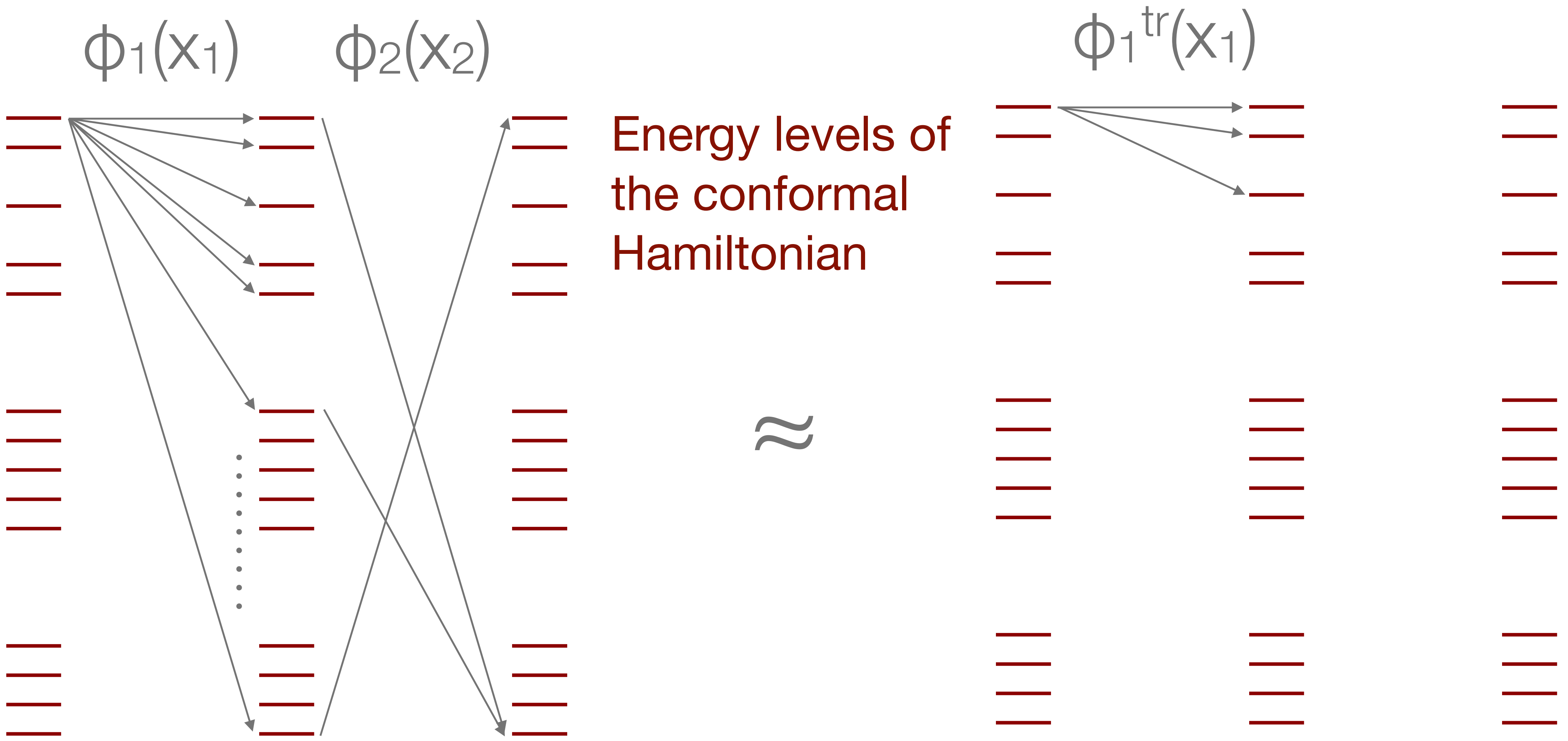

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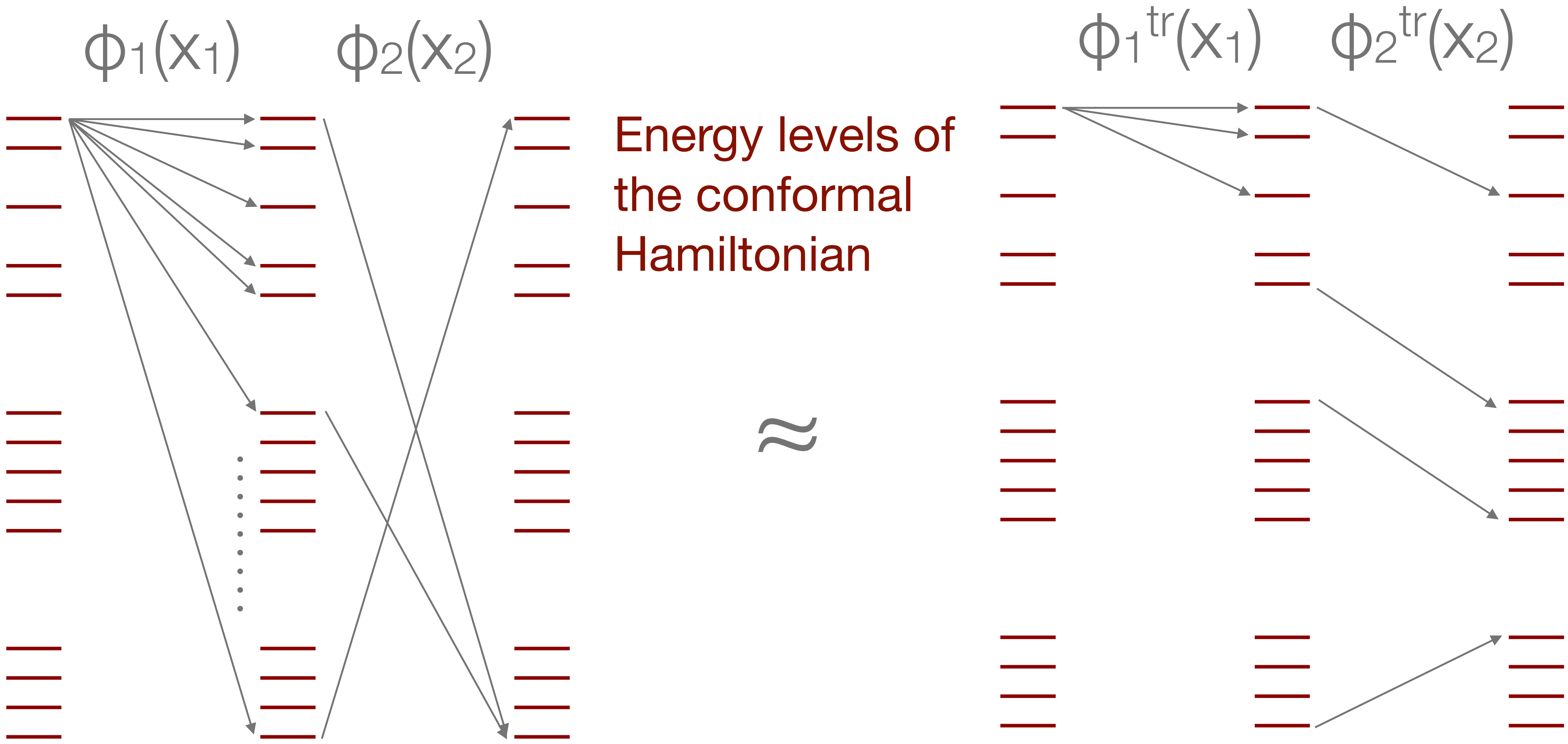

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- Our Approximations are constructive, provide rigorous error bounds and respect additional symmetries (WZW)
- Can be analysed further to understand low energy states of CFTs in quantum information theoretic terms (connection to quantum error correction?)
- Generalisation to MERA (multiscale entanglement renormalization Ansatz) seems possible and may provide better parameter scaling