Compactly-supported Wannier functions, algebraic *K*-theory, and tensor network states

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field	p	class		$d=0$	$d=1$	$d=2$
$\mathbf{C}$	0	A		$\mathbf{Z}$	$\mathbf{Z}$	$2.\mathbf{Z}$
	1	AIII		0	$\mathbf{Z}$	$2.\mathbf{Z}$
$\mathbf{R}$	0	AI		$\mathbf{Z}$	$\mathbf{Z}$	$\mathbf{Z}$
	1	BDI		$\mathbf{Z}/2$	$\mathbf{Z}/2 \oplus \mathbf{Z}$	$\mathbf{Z}/2 \oplus 2.\mathbf{Z}$
	2	D		$\mathbf{Z}/2$	$2.\mathbf{Z}/2$	$3.\mathbf{Z}/2 \oplus \mathbf{Z}$
	3	DIII		0	$\mathbf{Z}/2$	$3.\mathbf{Z}/2$
	4	AII		$\mathbf{Z}$	$\mathbf{Z}$	$\mathbf{Z}/2 \oplus \mathbf{Z}$
	5	CII		0	$\mathbf{Z}$	$2.\mathbf{Z}$
	6	C		0	0	$\mathbf{Z}$
	7	CI		0	0	0

Kitaev (2009);
Schnyder et al (2008)

“Tenfold way” classification of topological classes of band structures in various symmetry classes, based on topological *K*-theory of vector bundles [i.e. Atiyah’s $K^{-p}(T^d)$ and $KR^{-p}(T^d)$ groups] for torii T^d, up to $d=2$.

Z = group of integers, **Z**/2 = integers mod 2, *k*.**Z** = sum of *k* copies of **Z**

field	p	class	$\pi_0 K_0(\varphi_p^{(d)})$	$d=0$	$d=1$	$d=2$
$\mathbf{C}$	0	A	$\mathbf{Z}$	$\mathbf{Z}$	$\mathbf{Z}$	$2.\mathbf{Z}$
	1	AIII	$d.\mathbf{Z}$	0	$\mathbf{Z}$	$2.\mathbf{Z}$
$\mathbf{R}$	0	AI	$\mathbf{Z}$	$\mathbf{Z}$	$\mathbf{Z}$	$\mathbf{Z}$
	1	BDI	$\mathbf{Z}/2 \oplus d.\mathbf{Z}$	$\mathbf{Z}/2$	$\mathbf{Z}/2 \oplus \mathbf{Z}$	$\mathbf{Z}/2 \oplus 2.\mathbf{Z}$
	2	D	$(d+1).\mathbf{Z}/2$	$\mathbf{Z}/2$	$2.\mathbf{Z}/2$	$3.\mathbf{Z}/2 \oplus \mathbf{Z}$
	3	DIII	$d.\mathbf{Z}/2$	0	$\mathbf{Z}/2$	$3.\mathbf{Z}/2$
	4	AII	$\mathbf{Z}$	$\mathbf{Z}$	$\mathbf{Z}$	$\mathbf{Z}/2 \oplus \mathbf{Z}$
	5	CII	$d.\mathbf{Z}$	0	$\mathbf{Z}$	$2.\mathbf{Z}$
	6	C	0	0	0	$\mathbf{Z}$
	7	CI	0	0	0	0

N.R. (2016)

Table 1: Table of results for topological phases that can be realized using compactly-supported Wannier functions (polynomial sections) or TNSs. First three columns: labels for symmetry classes of topological phases. Fourth column: results of the analysis of the present paper for what can be realized with polynomial sections in dimension d, up to homotopy. Fifth through seventh columns: topological phases in general non-interacting fermion systems in dimensions $d=0$, 1, and 2, classified by $K^{-p}(T^d)$ (for $\mathbf{C}$) or $KR^{-p}(T^d)$ (for $\mathbf{R}$), for comparison with the fourth column.

Kitaev (2009)