Effects of spin-orbit coupling on the BKT transition and the vortex-antivortex structure in 2D Fermi Gases

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QMath13
Mathematical Results in Quantum Physics
Atlanta: October 10th, 2016
Main References for Talk

Ultra-cold fermions in the flatland: evolution from BCS to Bose superfluidity in two-dimensions with spin-orbit and Zeeman fields

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(Dated: June 22, 2012)

arXiv:1206.4984v1 (UNPUBLISHED)

Effects of Spin-Orbit Coupling on the Berezinskii-Kosterlitz-Thouless Transition and the Vortex-Antivortex Structure in Two-Dimensional Fermi Gases

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(Received 23 March 2014; published 17 October 2014)
Acknowledgements

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Outline

1) Introduction to 2D Fermi gases.

2) Creation of artificial spin-orbit coupling (SOC).

3) Quantum phases and topological quantum phase transitions of 2D Fermi gases with SOC.

4) The BKT transition and the vortex-antivortex structure.

5) Conclusions
Conclusions in words

• Ultra-cold fermions in the presence of spin-orbit and Zeeman fields are special systems that allow for the study of exciting new phases of matter, such as topological superfluids, with a high degree of accuracy.

• Topological quantum phase transitions emerge as function of Zeeman fields and binding energy for fixed spin-orbit coupling.
Conclusions in words

- The critical temperature of the BKT transition as a function of pair binding energy is affected by the presence of spin-orbit effects and Zeeman fields. While the Zeeman field tends to reduce the critical temperature, SOC tends to stabilize it by introducing a triplet component in the superfluid order parameter.

- In the presence of a generic SOC the sound velocity in the superfluid state is anisotropic and becomes a sensitive probe of the proximity to topological quantum phase transitions. The vortex and antivortex shapes are also affected by the SOC and acquire a corresponding anisotropy.
Conclusions in Pictures

TRANSITION FROM GAPLESS TO GAPPED SUPERFLUID

Change in topology
BKT transition and vortex-antivortex structure

Rashba

ERD
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Condensed Matter meets Atomic Physics

In optical lattices many types of atoms can be loaded like bosonic, Sodium-23, Potassium-39, Rubidium-87, or Cesium-133; and fermionic Lithium-6, Potassium-40, Strontium-87, etc...

In real crystals electrons or holes (absence of electrons) may be responsible for many “electronic” phases of condensed matter physics, such as metallic, insulating, superconducting, ferromagnetic, anti-ferromagnetic, etc...
How atoms are trapped?

- Atom-laser interaction
- Induced dipole moment.
- Trapping potential

\[ V(r, t) = -d \cdot E(r, t) \]
\[ d = -\alpha(\omega)E(r, t) \]
\[ V(r, t) = -\alpha(\omega)[E(r, t)]^2 \]
Atoms in optical lattices

\[ V(r) = -\alpha(\omega) \langle |E(r, t)|^2 \rangle \]
\[ V(r) = -\frac{1}{2} \alpha(\omega) [E(r)]^2 \]
How optical lattices are created?

- **a** Laser beam
- **b** Laser standing wave
- **c** Potential well
- **d** How particles are distributed in the lattice
Single plane excitations

Vortex-antivortex pairs

BKT transition:
Physics of 2D XY model
Critical Temperature

Pairing Temperature

Fermi Liquid

BCS-Bose Superfluidity in 2D

Bose Liquid 0.125
2D Fermi gases with increasing attractive interactions, but no SOC.
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Raman process and spin-orbit coupling

\[ \left( \frac{(k - k_R)^2}{2m} + \frac{\delta}{2} \right) \left( \frac{\Omega}{2} - \frac{(k + k_R)^2}{2m} - \frac{\delta}{2} \right) \]

\[ \hbar \omega_Z \]

\[ | -1 \rangle = | \downarrow \rangle \]

\[ | 0 \rangle = | \uparrow \rangle \]

\[ \delta/2 \]

\[ \delta/2 \]
SU(2) rotation to new spin basis:

$$\sigma_x \rightarrow \sigma_z ; \quad \sigma_z \rightarrow \sigma_y ; \quad \sigma_y \rightarrow \sigma_x$$

$$\begin{pmatrix}
\frac{k^2 + k_R^2}{2m} + \frac{\Omega}{2} & \quad -i \left( \frac{\delta}{2} - \frac{k_R}{m} k_x \right) \\
- \frac{\delta}{2} - \frac{k_R}{m} k_x & \quad \frac{k^2 + k_R^2}{2m} - \frac{\Omega}{2}
\end{pmatrix}$$

Geometry

\[ \hbar \omega_z \]

\[ |-1\rangle = |\downarrow\rangle \]

\[ |0\rangle = |\uparrow\rangle \]
Spin–orbit–coupled Bose–Einstein condensates

Y.-J. Lin¹, K. Jiménez-Garcia¹,² & I. B. Spielman¹

\[
\begin{pmatrix}
\frac{k^2 + k_R^2}{2m} + \frac{\Omega}{2} \\
\frac{\delta}{2} - \frac{k_R}{m} k_x \\
i\left(\frac{\delta}{2} - \frac{k_R}{m} k_x\right)
\end{pmatrix}
- i\left(\frac{\delta}{2} - \frac{k_R}{m} k_x\right)
\begin{pmatrix}
\frac{k^2 + k_R^2}{2m} - \frac{\Omega}{2}
\end{pmatrix}
\]

spin-orbit
detuning
Raman coupling
Hamiltonian with spin-orbit

\[ H = \sum_{k,s} \varepsilon(k)c_{ks}^+ c_{ks} - \sum_{k,s} h_{s's}(k)c_{ks}^+ c_{ks} \]
Parallel and perpendicular fields

\[ h_{\parallel}(k) = h_z(k) \]
\[ h_{\perp}(k) = h_x(k) - ih_y(k) \]

\[ H_0(k) = \begin{pmatrix}
\varepsilon(k) - h_{\parallel}(k) & -h_{\perp}(k) \\
-h^*_\perp(k) & \varepsilon(k) + h_{\parallel}(k)
\end{pmatrix} \]
Hamiltonian in terms of $k$-dependent magnetic fields

Hamiltonian Matrix

$$H_0(k) = \varepsilon(k)1 - h_x(k)\sigma_x - h_y(k)\sigma_y - h_z(k)\sigma_z$$

Momentum Space Two - Level System

in a momentum dependent magnetic field

$$h(k) = [h_x(k), h_y(k), h_z(k)]$$
Eigenvalues

$$\varepsilon_{\uparrow}(k) = \varepsilon(k) - |h_{\text{eff}}(k)|$$

$$\varepsilon_{\downarrow}(k) = \varepsilon(k) + |h_{\text{eff}}(k)|$$

$$|h_{\text{eff}}(k)| = \sqrt{|h_x(k)|^2 + |h_y(k)|^2 + |h_z(k)|^2}$$
Rashba Spin-Orbit Coupling

\[ H_R(k) = v_R \begin{pmatrix} 0 & k_y + i k_x \\ k_y - i k_x & 0 \end{pmatrix} \]
Equal-Rashba-Dresselhaus (ERD) Spin-Orbit Coupling

\[ H_R(k) = v_R \begin{pmatrix} 0 & k_y + ik_x \\ k_y - ik_x & 0 \end{pmatrix} \]

\[ H_D(k) = -v_D \begin{pmatrix} 0 & k_y - ik_x \\ k_y + ik_x & 0 \end{pmatrix} \]

\[ H_{ERD}(k) = v \begin{pmatrix} 0 & ik_x \\ -ik_x & 0 \end{pmatrix} \]
Energy Dispersions in the ERD case

Simpler case:

\[ h_x(k) = 0 \]
\[ h_y(k) = \nu k_x \]
\[ h_z(k) = 0 \]
\[ \varepsilon_{\uparrow}(k) = \varepsilon(k) - |\nu k_x| \]
\[ \varepsilon_{\downarrow}(k) = \varepsilon(k) + |\nu k_x| \]

\[ \varepsilon(k) = \frac{k^2}{2m} \]
Energy Dispersions and Fermi Surfaces

\[ \varepsilon_\alpha(k) = k^2 / (2m) \pm |v k_x| \]
Momentum Distribution (Parity)

\[
\begin{align*}
h_x(k) &= 0 \\
\frac{h_y(k)}{\varepsilon_F} &= 0.71 \frac{k_x}{k_F} \\
\frac{h_z(k)}{\varepsilon_F} &= 0.05
\end{align*}
\]
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Bring Interactions Back (real space)

\[ \mathcal{H}(r) = \mathcal{H}_0(r) + \mathcal{H}_I(r) \]

\[ \mathcal{H}_0(r) = \sum_{\alpha \beta} \psi_\alpha^\dagger(r) \left[ \hat{K}_\alpha \delta_{\alpha \beta} - h_i(r) \sigma_i,_{\alpha \beta} \right] \psi_\beta(r) \]

Kinetic Energy \quad Spin-orbit and Zeeman

\[ \mathcal{H}_I(r) = -g \psi_\uparrow^\dagger(r) \psi_\downarrow^\dagger(r) \psi_\downarrow(r) \psi_\uparrow(r) \]

Contact Interaction
Bring Interactions Back  
(momentum space)

\[ \mathcal{H}_I = -g \sum_q b^\dagger(q)b(q) \]

\[ b^\dagger(q) = \sum_k \psi^\dagger(k + q/2)\psi^\dagger(-k + q/2) \]

\[ \Delta_0 = -g \langle b(q = 0) \rangle \quad \text{and} \quad \Delta^*_0 = -g \langle b^+(q = 0) \rangle \]
Bring interactions back:
Hamiltonian in initial spin basis

\[ \tilde{K}_s(k) = \varepsilon(k) - \mu - sh_z(k) \]
Bring interactions back: Hamiltonian in the generalized helicity basis

\[
\tilde{H}_0 = \begin{pmatrix}
\xi^{\uparrow}(k) & 0 & \Delta_T(k)e^{-i\varphi_k} & -\Delta_S(k) \\
0 & \xi^{\downarrow}(k) & -\Delta^*(S)(k) & \Delta_S(k) \\
\Delta_T^*(k)e^{i\varphi_k} & -\Delta^*(S)(k) & -\xi^{\uparrow}(k) & 0 \\
\Delta_S^*(k) & -\Delta^*_T(k)e^{-i\varphi_k} & 0 & -\xi^{\downarrow}(k)
\end{pmatrix}
\]

\[
\varphi_k = \text{Arg} \left[ h_{\perp}(k) \right]
\]
Order Parameter: Singlet & Triplet

\[ \Delta_S(k) = \Delta_0 \frac{h_{\parallel}(k)}{|h_{\text{eff}}(k)|} \]

\[ \Delta_T(k) = \Delta_0 \frac{h_{\perp}(k)}{|h_{\text{eff}}(k)|} \]

\[ |\Delta_T(k)|^2 + |\Delta_S(k)|^2 = |\Delta_0|^2 \]

\[ h_{\perp}(k) = \nu k_x \quad h_z(k) = h_z \]

\[ h_{\text{eff}}(k) = (0, \nu k_x, h_z) \]

\[ h_{\text{eff}}(k) = \sqrt{\left|\nu k_x\right|^2 + h_z^2} \]
Excitation Spectrum

\[ E_1(k) = \sqrt{\left[ \left( \frac{\xi_{\uparrow} - \xi_{\downarrow}}{2} \right)^2 + \left( \frac{\xi_{\uparrow} + \xi_{\downarrow}}{2} \right)^2 + |\Delta_S(k)|^2 \right]^2 + |\Delta_T(k)|^2}, \]

\[ E_2(k) = \sqrt{\left[ \left( \frac{\xi_{\uparrow} - \xi_{\downarrow}}{2} \right)^2 + \left( \frac{\xi_{\uparrow} + \xi_{\downarrow}}{2} \right)^2 + |\Delta_S(k)|^2 \right]^2 + |\Delta_T(k)|^2}, \]

\[ E_3(k) = -E_2(k) \]

\[ E_4(k) = -E_1(k) \]

\[ \xi_{\uparrow}(k) = \mathcal{K}_+(k) - |h_{\text{eff}}(k)|, \]

\[ \xi_{\downarrow}(k) = \mathcal{K}_+(k) + |h_{\text{eff}}(k)|, \]

Can be zero
Excitation Spectrum

Making singlet and triplet sectors explicit

\[ E_2(k) \leftrightarrow E_-(k) \quad E_1(k) \leftrightarrow E_+(k) \]

\[ E_{p\pm}(k) = \sqrt{(E_S(k) \pm |h_{\text{eff}}(k)|)^2 + |\Delta_T(k)|^2}. \]

\[ E_S(k) = \sqrt{|K(k)|^2 + |\tilde{\Delta}_S(k)|^2}. \]

singlet sector
Excitation Spectrum (ERD)

\[ \Delta_T(k) = \Delta_0 \left| h_{\perp}(k) \right| / \left| h_{\text{eff}}(k) \right| = 0 \]
Lifshitz transition

Change in topology
Topological invariant (charge) in 2D

\[ \hat{m}(k) = (m_x, m_y) \]

\[ N_w = (2\pi)^{-1} \oint d\ell \hat{z} \cdot \hat{m} \times d\hat{m}/d\ell \]

\[ m_x(k) = \left[ E_S(k) - |h_{\text{eff}}(k)| \right] / E_{p-}(k) \]

\[ m_y(k) = \Delta_T(k) / E_{p-}(k) \]
Vortices and Anti-vortices of $m(k)$

$h_z / \varepsilon_F = 0.2$
$US - 0$

$E_b / \varepsilon_F = 1.0$

$h_z / \varepsilon_F = 1.5$
$US - 1$
For $T = 0$ phase diagram need chemical potential and order parameter

$$
\Omega_0 = V \frac{|\Delta_0|^2}{g} - \frac{T}{2} \sum_{k,j} \ln \{1 + \exp \left[-E_j(k)/T\right]\} + \sum_k \tilde{K}_+,
$$

$$
\tilde{K}_+ = \left[\tilde{K}_\uparrow(-k) + \tilde{K}_\downarrow(-k)\right] / 2
$$

$$
\frac{\delta \Omega_0}{\delta \Delta_0} = 0
$$

Order Parameter Equation

$$
N_+ = -\frac{\partial \Omega_0}{\partial \mu_+} = 0
$$

Number Equation
T = 0 Phase Diagram in 2D
Momentum distributions in 2D

FIG. 3: (color online) The momentum distributions $n_s(k_x, k_y)$ for ERD SOC $v/v_F = 0.8$ and $E_b/\epsilon_F = 0.1$ at $T = 0$, where $s = \uparrow (\downarrow)$ for upper (lower) panels. (a)(d) i-US-0 phase with $h_z/\epsilon_F = 0.2$; (b)(e) US-2 phase with $h_z/\epsilon_F = 0.4$; (c)(f) US-1 phase with $h_z/\epsilon_F = 1.0$. The color coding varies continuously from purple ($n_s = 0$) to red ($n_s = 1$).
Thermodynamic signatures of topological transitions
$T = 0$ Thermodynamic Properties in 2D

- $E_b/\epsilon_F = 0.1$ (solid line)
- $E_b/\epsilon_F = 0.2$ (dashed line)
- $E_b/\epsilon_F = 0.5$ (dotted line)
- $E_b/\epsilon_F = 1.0$ (dot-dashed line)

ERD SOC $v/v_F = 0.8$
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Hamiltonian in Real Space

\[ \mathcal{H}(\mathbf{r}) = \mathcal{H}_0(\mathbf{r}) + \mathcal{H}_I(\mathbf{r}) \]

\[ \mathcal{H}_0(\mathbf{r}) = \sum_{\alpha \beta} \psi_\alpha^\dagger(\mathbf{r}) \left[ \hat{K}_\alpha \delta_{\alpha\beta} - h_i(\mathbf{r}) \sigma_i,_{\alpha\beta} \right] \psi_\beta(\mathbf{r}) \]

Kinetic Energy

Spin-orbit and Zeeman

\[ \mathcal{H}_I(\mathbf{r}) = -g \psi_\uparrow^\dagger(\mathbf{r}) \psi_\downarrow^\dagger(\mathbf{r}) \psi_\downarrow(\mathbf{r}) \psi_\uparrow(\mathbf{r}) \]

Contact Interaction

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Effective Action at finite $T$

\[ \psi_{r,s} \rightarrow \psi_{r,s} e^{i\theta_r/2} \]

\[ \Delta_r = |\Delta_r| e^{i\theta_r} \]

\[ S = -\frac{1}{2} \text{Tr} \left\{ \ln \left[ \beta \left( \frac{A_+}{D_-} \frac{D_+}{A_-^*} \right) \right] \right\} - \frac{\beta L^2 |\Delta|^2}{g} \]

\[ + \frac{\beta}{2} \sum_{k,s} (-i\omega_n + k^2 - \mu_s) + \frac{1}{8L^2} \int dr \sum_k [\nabla_r(\theta_r)]^2. \]
Effective Action at finite $T$

$$S = S_{sp} + S_{fl}$$

$$S_{sp} = -\frac{1}{2} \text{Tr} \{ \ln [\beta M_k(0,0)] \} + \frac{\beta}{2} \sum_{k,s} (-i\omega_n + k^2 - \mu_s) - \frac{\beta L^2 |\Delta|^2}{g}$$

$$S_{fl} = \frac{1}{2} \int dr \left( A \left( \frac{\partial \theta_r}{\partial \tau} \right)^2 + \sum_{\nu=\{x,y\}} \rho_{\nu\nu} \left( \frac{\partial \theta_r}{\partial \nu} \right)^2 \right)$$
BKT Transition Temperature

(a) $h_z=0.0 \, \nu_R=1.0$

(b) $h_z=0.2 \, \nu_R=1.0$

- $\nu=0$
- $\nu_D=0$
- $\nu_D=0.5$
- $\nu_D=1.0$
Beyond the Clogston Limit

![Graph showing T_BKT vs. h_z with different EB values.](image)
Full Finite Phase Diagram
Anisotropic speed of sound

\[ \tilde{\omega}^2 - (q_x \ q_y) \begin{pmatrix} \rho_{xx} & \rho_{xy} \\ \rho_{xy} & \rho_{yy} \end{pmatrix} \begin{pmatrix} q_x \\ q_y \end{pmatrix} = 0 \]
Vortex-Antivortex Structure
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Change in topology

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BKT transition and vortex-antivortex structure

Rashba ERD
THE END